Supplementary Materials to "On highly D-efficient designs with non-negatively correlated observations"

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This supporting material lists the examples of the best chemical balance weighing designs under D-optimality criterion found by simulated annealing algorithm. They are presented in tables. The list of them is given below. The caption of table contains the parameters of a design (the numer of observations n and the number of objects p), the arguments of simulated annealing algorithm (the numbers I and R, since the argument T, called temperature, is always equal to 1), the number of runs of SA algorithm and possibly the values of the parameter ρ when there were found different designs for different values of this parameter. In table, there are values of the lower bound for D-efficiency of the best design found by SA algorithm for some values of the parameter ρ and that design is given under the table of these values. To simplify the notation $-$ denotes -1 and $+$ represents 1.

Remark 1. *When some design is mentioned in the text of the paper and it has a mark, it is given together with that mark, for example, the design* $\mathbf{A}$ *given by (4).*

List of Tables

Table 1: $n = 7$, $p = 6$, $I = 25$, $R = 0.95$, 10 000 runs, $\rho \leqslant 1/18$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.01	0.9027563	0.02	0.8988004	0.03	0.8951711	0.04	0.8918285	0.05	0.8887393

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Table 2: $n = 7$, $p = 6$, $I = 25$, $R = 0.95$, 10 000 runs, $\rho \geqslant 1/18$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.06	0.8864996	0.2	0.8741776	0.5	0.8646204	0.8	0.8608528
0.07	0.8851776	0.3	0.8696920	0.6	0.8630469	0.9	0.8600574
0.1	0.8817611	0.4	0.8667267	0.7	0.8618267	0.99	0.8594567

$$\mathbf{A} = \begin{pmatrix} - & + & + & + & + & + \\ - & - & + & - & - & + \\ - & - & - & + & + & - \\ - & + & - & + & - & + \\ + & + & - & - & + & - \\ + & - & - & + & + & + \\ + & + & + & + & - & - \end{pmatrix}$$

Table 3: $n = 11$, $p = 8$, $I = 100$, $R = 0.99$, 1000 runs, $\rho \leqslant 0.027$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.01	0.9487918	0.02	0.9469840	0.027	0.9458761

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Table 4: $n = 11$, $p = 8$, $I = 100$, $R = 0.99$, 1000 runs, $\rho \geqslant 0.028$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.028	0.9457699	0.2	0.9378966	0.5	0.9348450	0.8	0.9338121
0.029	0.9456665	0.3	0.9363795	0.6	0.9344033	0.9	0.9336045
0.1	0.9408803	0.4	0.9354609	0.7	0.9340711	0.99	0.9334501

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Table 5: $n = 11$, $p = 9$, $I = 100$, $R = 0.99$, 1000 runs

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0	0.9406063	0.2	0.9295173	0.5	0.9265719	0.8	0.9255733
0.01	0.9391546	0.3	0.9280539	0.6	0.9261450	0.9	0.9253725
0.1	0.9323898	0.4	0.9271670	0.7	0.9258237	0.99	0.9252231

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Table 6: $n = 11$, $p = 10$, $I = 100$, $R = 0.99$, 1000 runs

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0	0.9324283	0.2	0.9143540	0.5	0.9092870	0.8	0.9075400
0.01	0.9301428	0.3	0.9118521	0.6	0.9085420	0.9	0.9071869
0.1	0.9191801	0.4	0.9103209	0.7	0.9079796	0.99	0.9069238

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Table 7: $n = 15$, $p = 10$, $I = 100$, $R = 0.99$, 1000 runs, $\rho \leqslant 0.11$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.01	0.9658124	0.05	0.9629359	0.1	0.9611907	0.11	0.9609549

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Table 8: $n = 15$, $p = 10$, $I = 100$, $R = 0.99$, 1000 runs, $\rho \geqslant 0.12$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.12	0.9607463	0.2	0.9596792	0.5	0.9583233	0.8	0.9578974
0.15	0.9602571	0.3	0.9589860	0.6	0.9581392	0.9	0.9578137
0.17	0.9599976	0.4	0.9585848	0.7	0.9580027	0.99	0.9577518

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Table 9: $n = 15$, $p = 11$, $I = 100$, $R = 0.99$, 1000 runs, $\rho \leqslant 0.03$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.01	0.9603924	0.02	0.9595086	0.03	0.9587931

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Table 10: $n = 15$, $p = 11$, $I = 100$, $R = 0.99$, 1000 runs, $\rho \geqslant 0.033$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.04	0.9582301	0.2	0.9548488	0.5	0.9535819	0.8	0.9531839
0.05	0.9577649	0.3	0.9542012	0.6	0.9534099	0.9	0.9531057
0.1	0.9562369	0.4	0.9538264	0.7	0.9532823	0.99	0.9530478

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Table 11: $n = 15$, $p = 12$, $I = 100$, $R = 0.99$, 1000 runs

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0	0.9568695	0.2	0.9495767	0.5	0.9481382	0.8	0.9476851
0.01	0.9556958	0.3	0.9488420	0.6	0.9479425	0.9	0.9475960
0.1	0.9511475	0.4	0.9484161	0.7	0.9477972	0.99	0.9475301

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Table 12: $n = 15$, $p = 13$, $I = 100$, $R = 0.99$, 1000 runs, $\rho \leqslant 0.1$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0	0.9499905	0.05	0.9452388	0.1	0.9431712

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Table 13: $n = 15$, $p = 13$, $I = 100$, $R = 0.99$, 1000 runs, $0.11 \leqslant \rho \leqslant 0.42$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.11	0.9429271	0.2	0.9415343	0.3	0.9407678	0.42	0.9402557

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Table 14: $n = 15$, $p = 13$, $I = 100$, $R = 0.99$, 1000 runs, $\rho \geqslant 0.43$

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0.43	0.9402249	0.5	0.9400738	0.7	0.9397925	0.9	0.9396265
0.45	0.9401776	0.6	0.9399123	0.8	0.9397000	0.99	0.9395721

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Table 15: $n = 15$, $p = 14$, $I = 100$, $R = 0.99$, 1000 runs

ρ	D*-eff	ρ	D*-eff	ρ	D*-eff	ρ	D*-eff
0	0.9437595	0.2	0.9380845	0.5	0.9369690	0.8	0.9366180
0.01	0.9428440	0.3	0.9375146	0.6	0.9368174	0.9	0.9365490
0.1	0.9393041	0.4	0.9371844	0.7	0.9367048	0.99	0.9364979

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